

The competition of autonomous vehicles and its influence on the traffic

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Agenda

- Definitions & a bit of theory
- My assumptions
- The method of the analysis
- Simulation results
- Conclusions

The Game

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- in our setting, there will be two players playing against (or maybe not) each other.

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- The pair (x, y) in each cell represents the payoff for Player A and Player B, respectively.

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- Continuous payoff space that is expressed by a continuous function of each player's actions.

Continuous Payoff Space

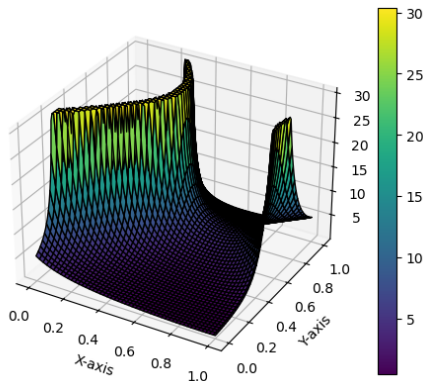
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- **Importance:**

- Provides a predictive tool for the outcome of strategic interactions.
- Widely applicable in economics, politics, and social sciences.

My Approach

In my simulation, I considered a two-player game on a fixed map. Each player chooses how many of his cars go either route. They alternate in those choices until they reach an agreement - convergence, or enough rounds pass - some scenarios do not converge at all.

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Players take turns planning where to go in the next "round". The player knows what his opponent decided on previously and reacts in an optimal way, considering the previous choice.

The second player makes a move considering either the current choice of his opponent or the previous one. This does not change the convergence of the system, even the speed of convergence remains roughly the same.

Time Function

$$t_i(q_i^x, q_i^y) = t_i^0 \cdot (1 + ((q_i^x + q_i^y)/Q_i)^2)$$

$$t(Q_x, Q_y) = \sum_i t_i(q_i^x, q_i^y) \text{ s.t. } \sum_i q_i^x = Q_x, \sum_i q_i^y = Q_y$$

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Explanation of Variables:

- Q_x : Fleet size of player 1
- Q_y : Fleet size of player 2
- Q_i : Capacity of road i (congestion parameter)
- t_i^0 : Time constant of road i
- q_i^z : Part of the fleet directed to the road i by the player z (flow)
- $t(Q_x, Q_y)$: Time of the travel for both players

Cost Function

From Time function:

$$t_i(q_i^x, q_i^y) = t_i^0 \cdot (1 + ((q_i^x + q_i^y)/Q_i)^2)$$

To the Cost function:

$$c_i(q_i^x | q_i^y) = q_i^x \cdot t_i(q_i^x, q_i^y) = q_i^x \cdot t_i^0 \cdot (1 + ((q_i^x + q_i^y)/Q_i)^2)$$

$$C(Q_x | Q_y) = \sum_i c_i(q_i^x | q_i^y) \text{ s.t. } \sum_i q_i^x = Q_x, \sum_i q_i^y = Q_y$$

Human-like objective: Minimizing the cost function

$$\min_{(\sum_i q_i^x = Q_x)} C((q_1^x, q_2^x, \dots, q_n^x) | Q_y)$$

Adding competition

$$P(Q_x, Q_y) = \alpha_x \cdot C(Q_x|Q_y) + \beta_x \cdot C(Q_y|Q_x)$$

Where:

- α_x : "Selfishness" of player X
- β_x : "Cooperativeness" of player X

For $\alpha > 0$ agent values his result.

With $\beta > 0$ agent wants to **cooperate**, cares also about opponent's result

With $\beta < 0$ agent seeks **competition**, he is rewarded for increasing his opponent's time

Cost Function - Simplified

In my approach

- There are only 2 routes, assuming they have equal capacity, we can replace constants t_i with one $t_{a/b}$ that reflects the ratio between speeds on both of them
- Since both fleets operate with unit fleets, instead of q_i^x we will consider q_a^x and $q_b^x = 1 - q_a^x$ as flows on each route
- Capacity of the road did not influence the results at all, so the problem simplified to exploring changes in q_a^x and q_a^y over time with given parameters α_x, β_x and α_y, β_y .

Despite those simplifications, an analytical solution is still not pretty, to say the least. I used numerical function optimizers since for convex functions they are accurate.

Behaviour Based on Parameters α and β

Definition:

- A "behaviour" is defined as a pair of parameters α and β .

Table of named Behaviours:

Behaviour	α	β
Human	1	0
Cooperator	1	0.99
Rival	1	-0.75
Altruist	0	1
Antagonist	0	-1

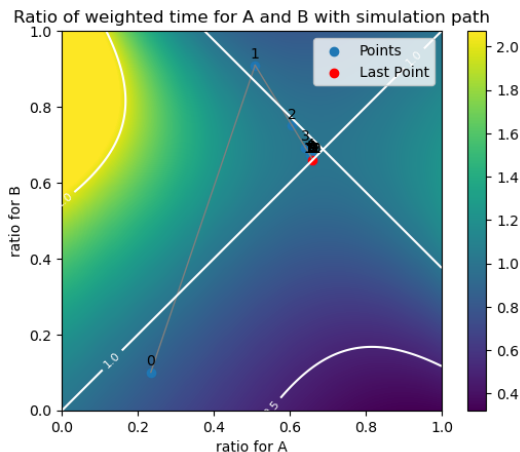
Explanation:

- Each row in the table represents a different behaviour, characterized by specific values of α and β .
- α : Represents the selfish factor.
- β : Represents the impact of other player's time on my reward function.

Turns in the Game

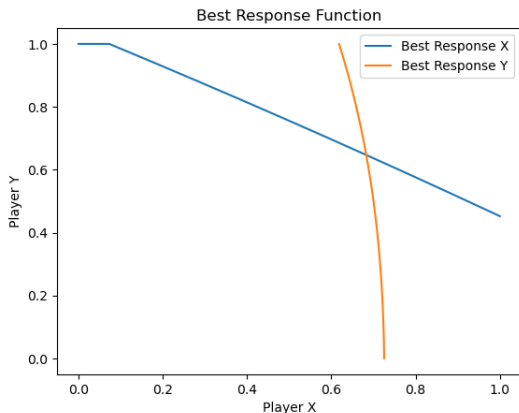
The player picks an optimal split for his fleet, finding the minimal cost available.

The full game might look like this:



Where to play next?

Good insight into the game trajectory can be achieved by plotting the "Best response" graph. Function $f(x)$ determines the best response for player X given the other player chose $(x, 1 - x)$. and function $g(y)$ does the same for player Y but with swapped axes.



Next steps

Starting from a random point, we can now see what the game will look like.

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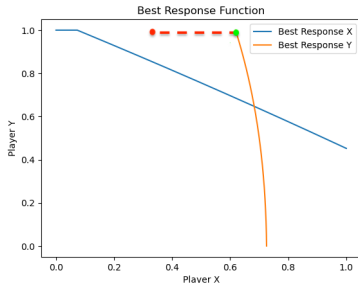


Figure: First point in our simulation

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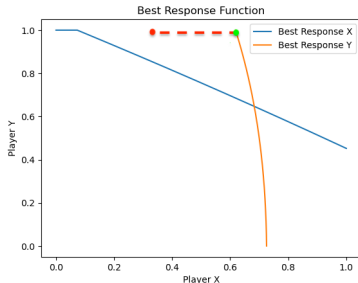


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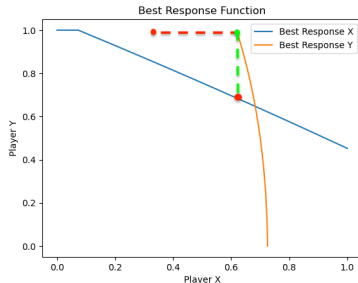


Figure: Second point in our simulation.

Next points are calculated in the same manner. Setting different values of behaviour parameters, we can achieve various trajectories of the game.

Game Trajectories

- The intersection point of best response functions is a Nash Equilibrium.
- Best response plot proves, that as long as plots intersect once, there exists exactly one Nash equilibrium, this is not always the case though.
- For example, the kind-kind game has infinitely many equilibria, it does finally converge to the symmetrical solution though.

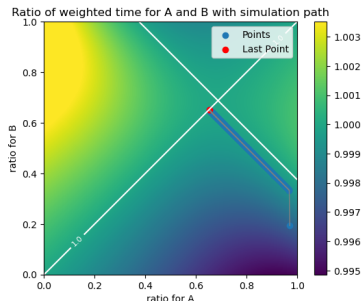
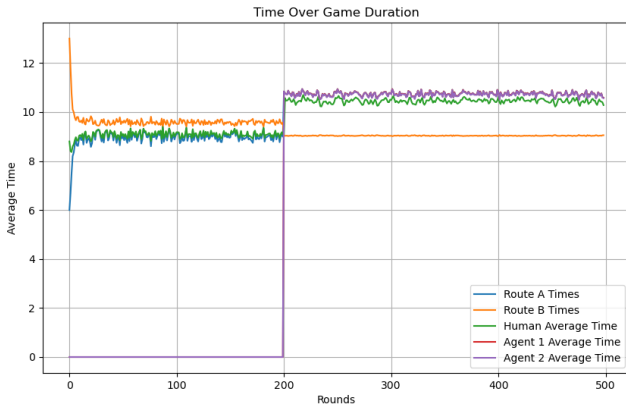


Figure: This game finally converged after 800 iterations

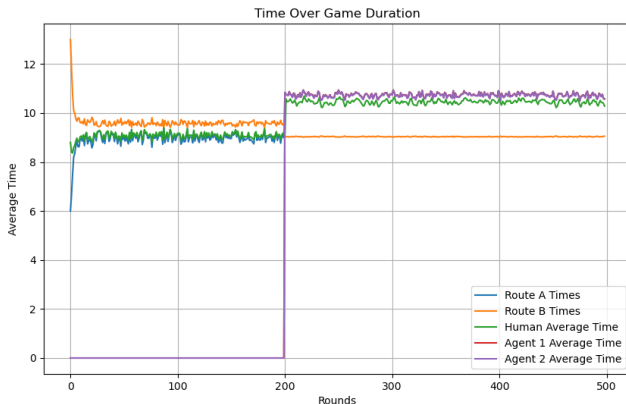
Adding the traffic

- In a pure one vs one scenario, the game is somehow predictable.
- One idea to make it more realistic and interesting is to start simulating normal traffic in our game.
- Agents simulating humans follow a Gawron algorithm for choosing their route each time.
- Firstly, for 200 iterations, humans are getting used to the system. After this time, I assign 40% of them to each fleet and I let the simulation run for 300 iterations more. The system stabilizes fast, and some changes are visible instantly.

Results - rivalry agents

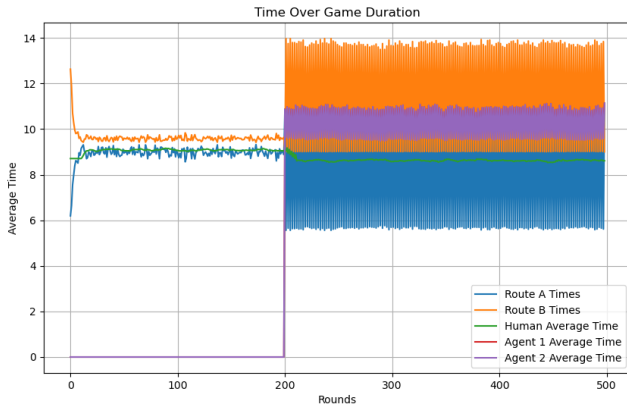


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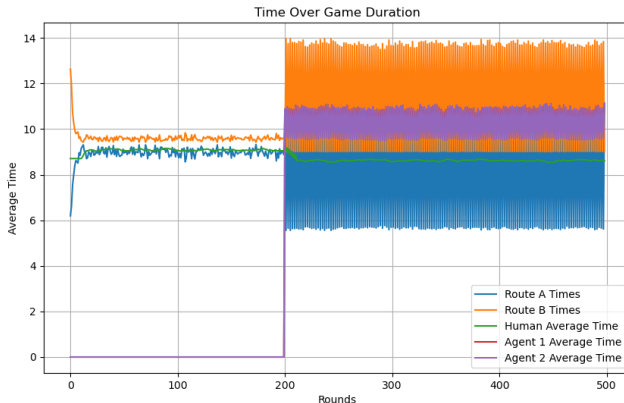


- System stabilized nearly instantly
- Average time, not only for humans, increased

Results - kind agents



Results - kind agents



- System did not achieve stability
- Average time for humans decreased

Acknowledgement



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